Wage Progression in Developed and Transition Economies

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Abstract

Life-cycle income growth rates differ significantly across countries. In this paper, we study the relative importance of human capital accumulation, labor market frictions and labor market opportunities in shaping life-cycle age-income profiles in Brazil, Colombia and the United States. We begin by documenting strong correlations between these three channels and the steepness of the life-cycle wage profiles across all three countries. Then, through the lens of a random search model with learning on the job, we find that labor market opportunities and labor market frictions can be as important as human capital accumulation in some countries when explaining the differences in the life-cycle wage profile.

Keywords: Labor markets, Income profiles, Developing countries
JEL codes: J24, J31, J63, J64, E24, O11, O12, O15

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1 Introduction

Workers across countries experience significant disparities in earnings per hour worked. Differences in wages, rent sharing, learning on the job, and job mobility all contribute to cross-country disparities in labor market outcomes and shape, over time, the ensuing age-income profile of workers.[1] In fact, some scholars have argued that life-cycle wage growth is potentially as important as schooling in explaining cross-country differences in labor productivity (Deming, 2023). To effectively assess the potential for enhancing worker outcomes through policy interventions, it is essential to discern the relative significance of forces shaping workers' age-earnings profiles.

In this paper, we examine the relative contribution of labor market frictions, human capital accumulation, and labor market opportunities in explaining cross-country variations in workers' age-income profiles. Our analysis focuses on the United States, Brazil, and Colombia, leveraging comprehensive data that reflect diverse stages of economic development. We employ longitudinal labor market data to track job-spells and compute wage growth for movers and stayers in three markets with different steepness in life-cycle wage profiles. We are particularly interested in Colombia and Brazil as recent research has shown a clear preponderance of small, old establishments (Eslava et al., 2022, 2021) that strongly contrast with the firm type distribution in the United States. These different environments can impact workers' age-income profiles, making them an important setting for study. Figure 1 depicts the age-income profiles, representing the average wage levels across different age groups in the three study countries.[2] Consistent with prior findings (Lagakos et al., 2018), the figure underscores a more pronounced growth trajectory of wages over a worker's lifespan in the United States compared to Brazil and Colombia.[3]

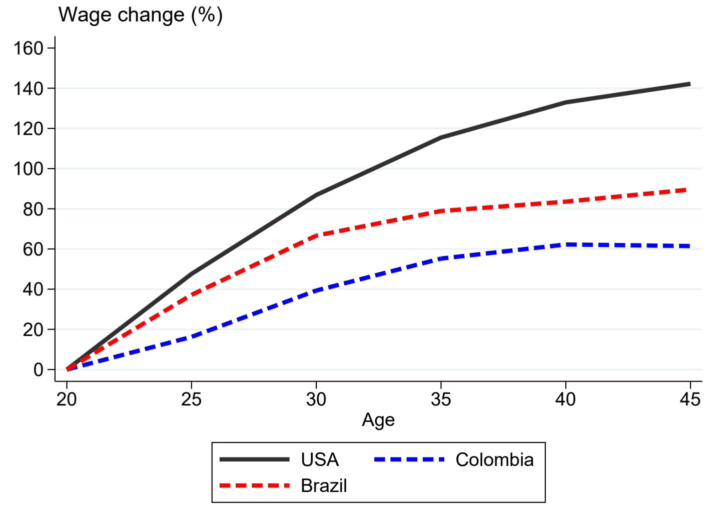
Labor market frictions, human capital accumulation, and labor market opportunities can jointly affect the age-income profile through three separate mechanisms. First, reduced on-the-job learning decreases average human capital growth, thereby lowering earnings growth throughout the life cycle directly. Second, heightened labor market fric-

[1]For the role of differences in wages, see: Lagakos et al. (2018). For differences in rent sharing, see Card et al. (2018). For differences of on-the-job training, see Ma et al. (2024) and Jedwab et al. (2023). For differences in job mobility, see Donovan et al. (2023) and Engbom (2022).

[2]Figure 1 employs data from the PSID for the United States, from the PNADC for Brazil, and from the social security records for Colombia (also known as the "Planilla Integrada de Liquidación de Aportes" (PILA)).

[3]Very similar patterns are observed when using alternative data sources and for the experience-income profile, as shown in Appendix Figures A.1 and A.2, respectively.

Figure 1: Age Profile of Wages



Notes: This figure presents growth rates of wages among workers with different age in different countries. Data for the USA comes from PSID, data for Brazil from PNADC, and data for Colombia from the social security records. Figure A.1 in the Appendix shows the same figure for all our counry data sets.

tions make it harder for workers to find higher-income positions and may contribute to more frequent setbacks in career advancement. Third, inadequate labor market opportunities prevent workers from accessing high-productivity jobs, thereby suppressing income growth through job-to-job mobility - a crucial driver of wage growth over the life cycle in developed economies (Topel and Ward, 1992; Hahn et al., 2021). Importantly, the significance of these factors is interconnected. For instance, low labor market frictions may have minimal impact on income growth if there is an insufficient number of high-quality firms in the economy. To the best of our knowledge, these factors have not been studied jointly in accounting for income growth differences across countries.

We analyze this question empirically and theoretically. First, we provide suggestive evidence showing that human capital growth might not be the only driver of differences in age-income profiles across countries. Second, we quantitatively assess the importance of the three channels using a random search model estimated on data from our three study countries. Our estimates confirm that labor market opportunities can pose a significant limit on wage growth over the life cycle.

We begin by presenting indicative empirical evidence suggesting that variations in learning-by-doing alone do not entirely account for the observed differences in income profiles across countries. Our empirical findings challenge the conventional notion that disparities in the age-income profile across countries are predominantly driven by human capital accumulation (Jedwab et al., 2023; Ma et al., 2024). This is generally evidenced by the fact that countries with higher average wage growth for job stayers exhibits higher relative wage growth across the life cycle. However, we complement these findings by establishing a positive correlation between wage growth for job movers, the proportion of individuals switching jobs, and life-cycle income growth. This implies that labor market frictions and the distribution of firm types play a significant role in shaping cross-country age-income profiles.

We then introduce a theoretical labor search framework to disentangle the roles of learning-by-doing, labor market frictions, and labor market opportunities, building upon the foundation laid by Burdett and Mortensen (1998) and Bagger et al. (2014). Our model reveals intricate interactions between human capital growth, labor market frictions, and labor market opportunities. For instance, higher wage growth rates decrease workers' reservation productivity and thereby extend the lower tail of the job ladder. They also reduce the steepness of the firm productivity-wage schedule. This implies that the benefits of higher on-the-job wage growth might be diminished by reduced average job market opportunities. Additionally, the presence of highly productive firms could coincide with increased market power at these firms, leading to a nuanced net effect dependent on various economic parameters, including labor market characteristics. As a result, the extent to which workers can benefit from a broad firm productivity distribution will depend on labor market frictions. Overall, these factors underscore the importance of considering a multifaceted interpretation of life-cycle income profiles. Both our empirical evidence and our theoretical model point to a complex understanding of the factors influencing wage growth patterns across countries.

We estimate the model using data for the United States, Colombia, and Brazil with the simulated method of moments. We rely on data moments accessible in most longitudinal labor force surveys, making them more readily available across countries compared to expensive and complex matched employer-employee data sets. While these moments reduce the precision of our perspective on the data, they are more useful for cross-country comparisons especially in a context of low-income countries. Specifically, we focus on the wage growth rate of job stayers, wage growth rate of job movers and the age-income profile, along with the unemployment and separation rates for estimation purposes. We harmonize the three data sets to reflect a common definition of work spells, and observe that, as an intermediate result, such reduction in heterogeneity compresses the cross-country differences in age-income profiles.

In a counterfactual analysis, we find that aligning the tail of the firm-type distribution with United States levels yields the most substantial positive impact on the life-cycle age profile for Colombia, while the Brazilian economy would benefit most from increases in on-the-job learning rates and labor market parameters at the United States' level. These findings resonate with the findings in Eslava et al. (2022) who also find that smaller and likely unproductive firms account for a larger employment share in Colombia as compared to the United States. Our results indicate that age-income disparities are not solely driven by one factor but rather result from the interplay of all three forces.

Our paper builds on three groups of literature. First and foremost, our paper relates to the work that links wage progression to the level of development in a country. In addition to outlining variations in income growth, these studies predominantly propose that disparities in human capital accumulation contribute to the differences observed among countries. Lagakos et al. (2018) document that wage growth in developed economies is on average twice as steep as growth rates in developing economies. Fang and Qiu (2022) find this same pattern when comparing China with the United States. Jedwab et al. (2023) observe very similar patterns when comparing workers in 145 countries, and suggest that workers accumulate more human capital on the job in developed economies. Related to this point, Ma et al. (2024) suggest that workers in developed economies are provided with more on-the-job training, and this can partially explain wage growth disparities. Guner et al. (2018) show that life cycle earnings growth of managers relative to non-managers is increasing in economic development and suggest that distortions disincentivize learning. Finally, Engbom (2022) documents that wage growth is greater in countries with more job-to-job mobility and explains 50% of cross-country age-income differences across OECD countries with differences in labor market fluidity alone, without considering differences in labor market opportunities. Donovan et al. (2023) find that both job-finding and employment-exit rates are negatively correlated with development, highlighting the importance of accounting for labor market frictions and human capital growth as well as labor market opportunities in explaining wage growth differences across countries.

Second, this paper contributes to the literature on random search labor models with the objective of explaining income profiles both within and across countries. While there exists a substantial body of literature explaining cross-country differences in mobility rates and labor market outcomes (e.g., Jolivet et al. (2006)), as well as explaining wage growth rates on-the-job (e.g., Rubinstein and Weiss (2006); Barlevy (2008); Yamaguchi (2010); Gregory (2020)), only a few studies focus on decomposing wage growth patterns. Notably, Bagger et al. (2014) and Menzio et al. (2012) decompose wage growth into human capital accumulation and job search using two distinct labor search frameworks. Building upon the former, Ozkan et al. (2023) show that human capital growth patterns are important determinants of income differences within the United States economy. Similarly, Burdett et al. (2011) suggest a model related to Burdett and Mortensen (1998) incorporating human capital accumulation but without firm heterogeneity and with production complementarity. However, unlike this paper, their aim is not to explain differences in wage growth rates within their framework. In contrast to Ma et al. (2024), we calibrate the model to the three study countries, thus enabling the decomposition of effects across all three dimensions.

Third, we contribute to the literature that highlights the role of productivity dispersion for development. This body of literature shows that the growth rate of manufacturing plants during the life cycle is lower in less-developed countries and that the variance of firm productivity within narrowly defined industries is higher in less developed countries (Hsieh and Klenow, 2009, 2014; Hsieh and Olken, 2014; Poschke, 2018). Specifically for Latin America, Eslava et al. (2021) address this topic by showing that the firm size distribution in the region exhibits a predominance of small businesses, and this is partly explained by a slower exit rate of smaller firms (Eslava et al., 2022). Our paper suggests that a compressed firm productivity distribution with less outstanding firms can contribute to the lower slope of the age-income distribution in developing countries.

The paper proceeds as follows. Section 2 summarizes our data sources. Section 3 presents motivating evidence based on cross-country data. Section 4 presents the theoretical framework and section 5 shows estimation results. Finally, Section 6 draws the conclusion.

2 Data

In this section we begin by introducing the data sources (section 2.1). We then describe the construction of key variables (section 2.2), and finish by providing some descriptive statistics of the different samples (section 2.3).

2.1 Data Sources

Our analysis is based on longitudinal and cross-sectional data on workers' mobility and wages. We employ diverse data sources that extensively cover labor markets over time in the United States, Brazil, and Colombia. We leverage the diverse strengths of the data sets to speak to different aspects of workers' labor market histories.

To examine the wage evolution over the life-cycle in the United States, we employ three alternative datasets. First, we use the "Panel Study of Income Dynamics" (PSID) spanning from 1975 to 2013.[4] The PSID is a longitudinal household survey initiated in 1968 and collected annually from 1975 to 1998, with biennial coverage after 2001. The PSID has been used in previous studies similar to ours, enabling us to compare our results to prior findings (see, for instance, Lagakos et al. (2018)), but it does not include information about job mobility. Therefore, we complement our analysis using the outgoing rotation group of the "Current Population Survey" (CPS) from 2003 to 2013.[5] Households in the CPS are initially interviewed and subsequently followed up at intervals of four, eight, twelve, and sixteen months after the initial survey. Due to its design, it is not feasible to track the same sample of workers for a period longer than a year and a half. Therefore, we also employ the 1997 wave of the "National Longitudinal Survey of Youth" (NLSY). The NLSY constitutes a nationally representative sample of individuals born between 1980 and 1984 in the United States, surveyed longitudinally over an extensive period of time. Our focus lies in the waves spanning 2000 to 2019, capturing the labor market evolution of individuals aged approximately 20 to 40 years during this period.

To examine the Brazilian labor market, we employ the 2012 to 2019 waves of the "Pesquisa Nacional por Amostra de Domicílios Contínua" (PNADC). This nationally representative survey, initiated in 2012, is designed to monitor the dynamics of Brazilian labor markets. Workforce indicators are collected quarterly, and selected household are interviewed for five consecutive quarters.

For Colombia, we rely on social security records encompassing the entire population of formal workers who held a formal job from 2009 to 2016.[6] We do not observe workers with informal jobs or those who are unemployed. Therefore, we complement our analysis

[4]We use the waves from 1975 to 2013 to ensure comparability with Lagakos et al. (2018).

[5]We use these years to be comparable with the PSID.

[6]Formal workers are defined as those who contribute to health and pension, constituting approximately 60 percent of jobs.

with Colombian household surveys from 2009 to 2016. These surveys include information on both formal and informal workers and are nationally representative. The data, derived from the "Gran Encuesta Integrada de Hogares" (GEIH), are officially compiled monthly to monitor household dynamics in the country. These data do not track workers longitudinally, but it is still possible to use them for computing age- and experience-income profiles (as illustrated in Appendix Figures A.1 and A.2).

We complement our analysis using the European Union Statistics on Income and Living Conditions (EU-SILC) data from 2005 to 2019. These data cover an extensive number of European countries (33), although they are not fully comparable to our previous datasets. The EU-SILC is a rotating yearly panel that surveys individuals in European countries over several periods. Most countries follow individuals for up to four years, but countries like France, Slovakia, Lithuania, and Luxembourg do so for eight years.[7] We use these data to create cross-country correlations in our measures of interest.

2.2 Key Variables

In all longitudinal data sets, we create measurements of wages, job mobility, and wage growth rates. We aim at harmonizing variable definitions as much as possible across all the alternative data sets. We focus on monthly wages in all data sets since we cannot compute wages per hour across all of them. These measures are deflated and expressed in U.S. Dollars to guarantee comparability. Additionally, we winsorize wages over the first and 99th percentile to deal with outliers in a consistent way.

Job mobility and wage growth rates are computed yearly across all samples. Individuals who work in the same job for one consecutive year are considered job stayers whereas job switchers are the complement of this situation. Due to the time differences in the gathering of the longitudinal waves across data sets, we homogenize wage growth and mobility rates to reflect yearly movements. For the CPS and the PNADC, a stayer is defined as a person who did not switch jobs during three consecutive quarters. For the PSID, however, it is not possible to properly identify job mobility. Therefore, we follow Lagakos et al. (2018) and identify job stayers as those who do not change occupation or industry for two consecutive years. Data in the PSID are collected annually between 1975 and 1998 and every two years between 2001 and 2013, therefore wage growth rates are

[7] We exclude Portugal, Germany, and Norway from our analysis. Portuguese data differ in how incomes were recorded, Norwegian data recorded job transitions differently, and German data were only available for two periods with a very small sample size.

halved for the period 2001 to 2013.[8] For the case of the NLSY, we cannot track individuals beyond age 39, but it enables accurate identification of labor mobility across employers.[9] For the Colombian social security records we are able to observe employers in every year, enabling us to compute job transitions. Finally, for the EU-SILC, individuals are asked if they switched employers in the last 12 months.

2.3 Sample

We base our analysis on male workers observed between the ages of 20 to 60 who have a full-time job. With the aim of reducing outliers in the population of full-time workers, we winsorize wage growth observations, separately for job movers and job stayers.

Table 1 provides a description of the main different datasets. Additionally, in Appendix Table A.1, we present a description of the samples with no sample restrictions except the age restriction. We observe similar average wage growth rates across all datasets, except for the NLSY, where we focus on young workers whose salaries grow faster. We also note that wage growth for job movers is typically higher than wage growth for job stayers, consistent with mobility being dominated by job-to-job changes. Furthermore, in contrast to the full sample described in Appendix Table A.1, we find that wage growth is more stable when focusing exclusively on males employed in full-time jobs.[10]

[8] To accommodate the gap between 1998 and 2001, we compute wage growth rates and job transitions every three years, dividing the wage growth rate by three.

[9] The NLSY was collected annually until 2011 and biennially thereafter. Consequently, the definition of a "stayer" varies slightly before and after 2011.

[10] The standard deviation of the wage growth for stayers and movers reduces considerably when comparing only males (in Table 1 to the full sample A.1.

Table 1: Summary Statistics

	Obs. (1)	Mean (2)	SD. (3)	Median (4)	Min. (5)	Max. (6)
A) USA (PSID)						
Age	61,526	39.52	10.45	39.00	20.00	60.00
Female	61,526	0.00	0.00	0.00	0.00	0.00
Log(Monthly Wages)	61,526	8.33	0.64	8.35	4.12	9.68
1(Stayer)	7,633	0.43	0.50	0.00	0.00	1.00
Wage Growth for Stayers	3,095	0.02	0.15	0.01	-0.69	0.75
Wage Growth for Mover	4,270	0.02	0.18	0.01	-0.74	0.79
B) USA (NLSY)						
Age	20,126	28.06	5.15	27.00	20.00	39.00
Female	20,126	0.00	0.00	0.00	0.00	0.00
Log(Monthly Wages)	20,126	7.97	0.56	7.93	4.77	9.23
1(Stayer)	7,583	0.92	0.28	1.00	0.00	1.00
Wage Growth for Stayers	6,871	0.05	0.20	0.03	-0.81	0.96
Wage Growth for Mover	712	0.07	0.48	0.04	-1.55	1.99
C) USA (CPS)						
Age	752,662	39.49	11.00	39.00	20.00	60.00
Female	752,662	0.00	0.00	0.00	0.00	0.00
Log(Monthly Wages)	752,662	8.08	0.64	8.08	5.58	9.35
1(Stayer)	186,666	0.95	0.21	1.00	0.00	1.00
Wage Growth for Stayers	178,118	0.02	0.44	0.00	-1.47	1.49
Wage Growth for Mover	8,548	0.03	0.50	0.01	-1.64	1.72
F) Brazil (PNADC)						
Age	2,815,852	38.02	10.74	37.00	20.00	60.00
Female	2,815,852	0.00	0.00	0.00	0.00	0.00
Log(Monthly Wages)	2,815,852	6.47	0.76	6.41	3.53	8.43
1(Stayer)	244,646	0.98	0.15	1.00	0.00	1.00
Wage Growth for Stayers	238,413	0.01	0.38	-0.03	-1.36	1.35
Wage Growth for Mover	6,233	0.02	0.48	0.01	-1.60	1.53
E) Colombia (Social Security)						
Age	12,310,369	42.13	9.61	42.00	20.00	60.00
Female	12,310,369	0.00	0.00	0.00	0.00	0.00
log(Monthly Wages)	12,310,369	6.07	0.66	5.80	-8.22	11.78
1(Stayer)	9,433,955	0.87	0.34	1.00	0.00	1.00
Wage Growth for Stayers	8,210,762	0.02	0.08	0.01	-0.50	0.50
Wage Growth for Mover	1,223,193	0.04	0.39	0.02	-1.00	1.00

Notes: All wages are expressed in 2010 USD, and winsorized in the 1st and 99th percentile. Colombian data have a yearly frequency. Brazilian data have quarterly frequency. The U.S. PSID data set is collected every two years, whereas the U.S. CPS is collected yearly. All samples are conditioned to full-time working males, between the ages of 20 to 60.

3 Empirical Motivation

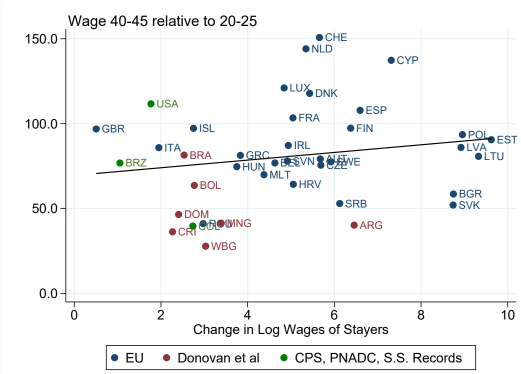
Cross-country differences in life-cycle income profiles are oftentimes attributed to differences in on-the-job human capital accumulation (Jedwab et al., 2023; Ma et al., 2024). However, labor market frictions and labor market opportunities are arguably also pivotal factors contributing to the differences in age-income profiles between developing and developed economies by varying the pool and quality of available jobs. All three factors play a role in shaping workers' ability to ascend the job ladder towards more productive and better paying jobs, thereby affecting the potential for income growth through job-to-job mobility. In fact, wage growth has been shown to be greater in more fluid labor markets and this relationship seems to be mediated by income gains in job-to-job transitions and its interaction with training and human capital (Engbom, 2022).

Building on this intuition, we provide two pieces of evidence leveraging cross-country and cross-regional variation to identify suggestive empirical evidence of the importance of the three channels in explaining variations in income profiles across countries. We draw from data on Brazil (PNADC), the United States (CPS), Colombia (Social Security records), and Europe (EU-SILC) to illustrate correlations between labor market frictions, human capital accumulation, and labor market opportunities with the relative wage of individuals possessing 20 years of potential labor market experience in comparison to labor market entrants. We collapse the data at the country or region level and compute a measure of relative wage growth as the ratio of wages of workers between the ages of 40 to 44 relative to those with 20 to 24 years or age. Human capital accumulation is captured by the wage growth of job stayers, while labor market frictions are proxied using the share of job switchers. To account for labor market opportunities, we use the wage growth of job switchers.

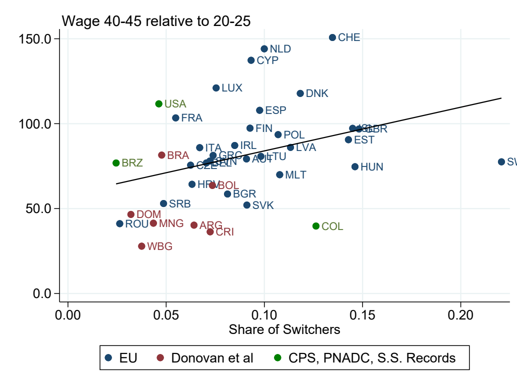
We first consider cross-country correlations of these three measures with the relative wage growth during the first 20 years of experience. The results are displayed in Figure 2. We find positive correlations between life-cycle wage growth and: 1) human capital accumulation, measured by the wage growth for job stayers (Panel 2a); 2) labor market frictions, measured by the share of job switchers (Panel 2b,); and 3) labor market opportunities, measured by the change in the wage of job switchers (panel 2c). These results challenge the conventional view that on-the-job learning is the major driver of cross-country differences in age-income profiles, although no direct causality is claimed. Human capital accumulation might not be the sole or primary factor explaining cross-country differences in life-cycle wage growth. Labor market frictions and opportunities could indeed play a fundamental role, as implied by the positive cross-country correlations shown in Panels 2b and 2c.

Figure 2: Cross-Country Correlation

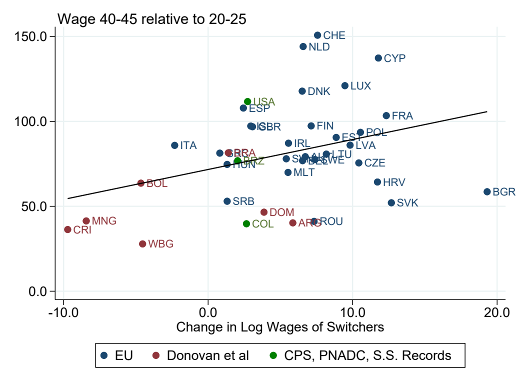
(a) Wage Growth of Job Stayers



(b) Share of Job Switchers



(c) Wage Growth of Job Switchers



Notes: These figures combine data from EU-SILC (blue markers), from Donovan et al. (2023) (red markers), and the from the CPS, PNADC, and Colombian social security records (green markers). The y-axis corresponds to the ratio of monthly wages paid to workers between 40-45 years old, relative to those with 20-25 years of age. The x-axis in Figure 2a corresponds to the log change in wages of stayers. The x-axis in Figure 2b corresponds to the share of job switchers. The x-axis in Figure 2c corresponds to the log change in wages of job switchers. Each point corresponds to a time invariant value computed first by collapsing individual-level data at the country-year level, and then averaged across years.

Cross-country correlations, however, may be susceptible to various biases, especially due to comparability issues arising from differences in sample composition. To address these concerns, we follow Engbom (2022) by computing individual-level Mincer regressions that control for time-invariant individual characteristics and for time trends, along with proxies for our three channels. We use worker-level panel data for the United States (CPS) and Brazil (PNADC), aggregating the channel variables at the country-state-year level.[11] Notably, we incorporate individual fixed effects to mitigate potential concerns related to sample selection. Formally, we estimate:

$$\ln w_{it} = \beta(D_{sc(it)} \times a_{it}) + \gamma X_{it} + \mu_i + \mu_t + A_{it} + \varepsilon_{it}, \tag{1}$$

where D_{sc} corresponds to four time-invariant measures computed at the country, c, and state, s, level: 1) the share of workers that switch jobs; 2) the percent change in wages of those workers; 3) the percentage change in wages for workers who switch firms; and 4) the share of workers employed in firms with more than 10 employees.[12] We include the share of workers in firms with more than 10 employees to also account for labor market opportunities.[13] We interact these measures with the worker's age at year t (a_{it}) and include individual (μ_i), year (μ_t), and age group (A_{it}) fixed effects.[14] Standard errors are clustered at the state level, and sample weights are re-scaled to give equal weight to each country used.[15]

We present the results of the estimation of Equation 1 through a binned scatter plot of its residuals in Figure 3. In Panel 3a, the average log wage of job stayers (computed at the country-by-state level) serves as the independent variable. Panel 3b explores the relationship with the share of job switchers, Panel 3c examines the connection with the wage growth of job switchers, and Panel 3d uses the share of workers employed in firms with more than 10 employees. Once again, wage growth for job stayers proxies for on-the-job human capital accumulation, the share of job switchers addresses labor market frictions, and the wage growth of switchers and the share of workers employed in big firms reflect labor market opportunities. As a robustness check, we conduct a similar analysis for 29 European countries using data from EU-SILC. We use cross-country variation, rather

[11]We exclude our Colombian data from this exercise as it only covers the formal sector of the economy, which represents around 60 percent of jobs in Colombia. In our later analysis, we restrict attention to a comparable set of jobs across countries. We provide alternative estimations for European countries as a robustness analysis.

[12]These measures are computed by first collapsing the individual-level measures at the year, age group, country, and state level. Then, we compute the time-invariant average across years at the country-state level.

[13]In the standard Burdett and Mortensen (1998) model, firm size increases in firm productivity such that the share of workers in firms of a particular size is an indicator of the tail of the job distribution.

[14]Age group fixed effects corresponds to categorical variables for age groups 20-24, 25-29, 30-34, 35-39, 40-41, 45-49, 50-54, 55-60. Workers outside of this range are excluded of the regression.

[15]We also present cross-regional correlations in the Brazilian (PNADC) and US (CPS) data in Appendix Figure A.3. We observe positive correlations for all measures except for the change in the wages of stayers.

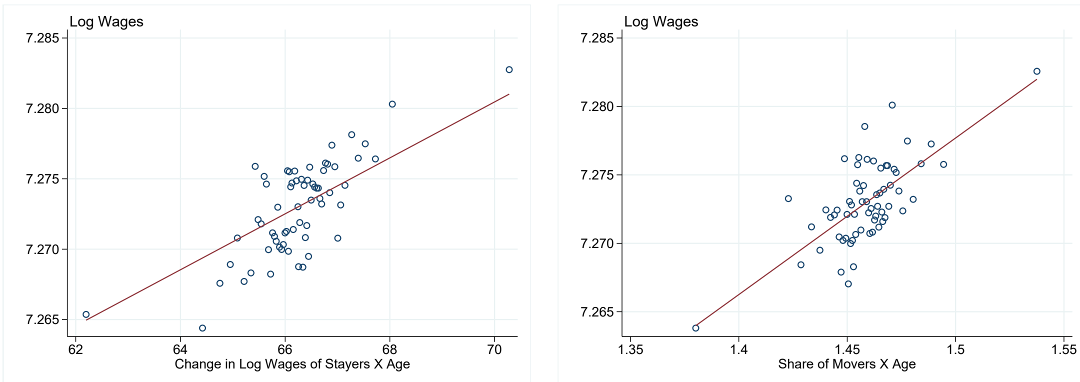
than state variation, and estimate the same specification as in Equation 1. We present the results in Appendix Figure A.4. We find very similar patterns using this data set.

All measures exhibit positive correlations with log wages, indicating that areas with higher on-the-job human capital accumulation, more fluid labor markets, and better labor market opportunities tend to have larger life-cycle wage growth.[16] Notably, the relationship between wage growth for job stayers (representing on-the-job human capital accumulation) exhibits a positive and robust correlation, aligning with previous findings. Nonetheless, all other three measures present a similar positive pattern. This implies that, while on-the-job human capital accumulation seems to contribute to differences in life-cycle wage growth, other factors also play a role in such disparities. The empirical evidence presented herein suggests that labor market frictions and opportunities could be as crucial as human capital accumulation in explaining life-cycle wage growth.

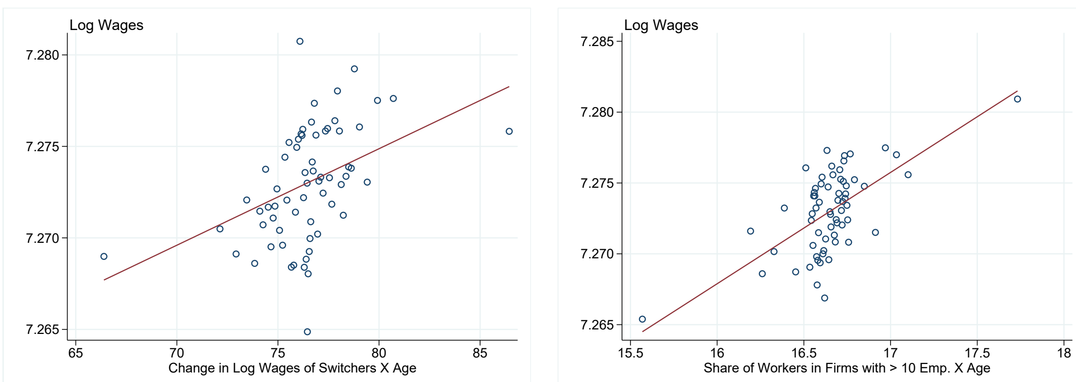
[16]The estimation of Equation 1 is available in Appendix Table A.2, indicating a positive and statistically significant correlation between each channel and life-cycle wage growth.

Figure 3: Determinants of Life-Cycle Wage Growth

(a) Wage Growth of Job Stayers (b) Share of Job Switchers



(c) Wage Growth of Job Switchers (d) Share of Employment in Big Firms



Notes: These figures display binned scatter plots using the log of monthly wages as outcome in the y-axis and the wage growth of job stayers (Panel 3a), the share of job switchers (Panel 3b), the wage growth of job switchers (Panel 3c). The independent variables are computed as state-by-country averages, and are interacted with the individual's age. All estimations include individual, year, and age group fixed effects, and standard errors clustered at the state-by-country level.

4 Framework

To quantitatively assess the relative significance of worker learning and the job ladder in shaping wage growth patterns, we present a parsimonious random search model. The model follows Burdett and Mortensen (1998) but incorporates worker heterogeneity, learning and wage piece-rates. In its timing convention regarding the arrival of events, we follows Bagger et al. (2014). We start by outlining the model setting in section 4.1 and then present value functions in section 4.2. We solve the problem of the firm in section 4.3 to derive the wage equation.

4.1 Setting

The economy is populated by a continuum of workers of size M and a mass of firms of size L. Time is discrete, and workers and firms discount the future at a common rate $\tilde{\beta}$. Each period a measure of workers ν dies and an equivalent measure of worker is reborn. Denote with $\beta = (1-\nu)\tilde{\beta}$ the aggregate discount rate. Workers and firms have heterogeneous productivities, indexed by p for firms and h for workers. Workers can be employed or unemployed. When unemployed, they have a chance of meeting a new firm at Poisson rate λ, and when employed, they face the exogeneous probability of match dissolution at rate δ. Workers are endowed with an initial level of skills $h_0 \geq 0$ that grows, when employed, at rate μ. The skills of employed workers then follows the law of motion:

$$h' = h + \mu.$$

Workers are entitled to receive unemployment benefits of the form:

$$b(h) = b_0 + h.$$

Firm productivity is distributed according to the distribution $\Gamma(p) \in [\underline{p}, \infty]$. We denote its anti-cumultative distribution function as $\bar{\Gamma}(p) = 1 - \Gamma(p)$. Output y of the worker-firm match is simply additive in the two productivities of the worker-firm match:

$$y = p + h.$$

Firms post wages, $w_0(p)$, that are a piece-rate, $r(p)$, of productivity p of the firm, such that:

$$w(p,h) = r(p) + p + h = w_0(p) + h \tag{2}$$

The endogenous distribution of wages is denoted $F(w)$, such that $w \in [\underline{w},\overline{w}]$ with anti-cumulative distribution function $\bar{F}(x) = 1 - F(x)$.

4.2 Value Functions

Let $U(h)$ denote the value of an unemployed worker with productivity h, and let $W(w_0, h)$ denote the value of an employed worker with skills h at a firm offering w_0. The utility of an unemployed worker is composed of the flow value of unemployment $b(h)$ and the option value of firm matching as well as skill depreciation. When matching with a firm at rate λ, the worker is promised utility $W(w_0(p), h)$ and moves to the new job if the value at the job exceeds the value in unemployment. We can hence write the worker's lifetime value in unemployment as follows:

$$U(h) = b(h) + \beta(1-\lambda)U(h) + \beta\lambda \int \max\{U(h), W(w_0, h)\} dF(w_0).$$

When employed, the worker receives wages w and faces the option value of learning and mobility to other jobs as well as the option value of unemployment due to displacement. At exogenous Poisson rate δ, the worker is displaced from his current job and becomes unemployed, yielding the value $U(h)$. When the worker is not displaced, he learns, increasing his human capital to value h'. He has then the opportunity to meet another firm. At Poisson rate λ, the worker meets an outside firm and decides whether to move to the new firm or stay with the incumbent firm. The worker's value when employed is then:

$$\begin{aligned} W(w_0, h) &= w + \beta\delta U(h) + \beta\lambda \int \max\{W(w_0, h'), W(x, h')\} dF(x) \\ &+ \beta(1-\delta-\lambda) W(w_0, h'). \end{aligned}$$

As standard in this type of model, there exists a reservation wage component $\theta^R = w^R(p, h) - h$ such that an unemployed worker will accept the current job offer, that is $W(\theta^R, h) = U(h)$. We guess and verify an equilibrium in which the reservation wage component is identical for all workers and show its derivation in Appendix section B.1. We obtain the reservation component as

$$\theta^R = b_0 - \mu \frac{\beta(1-\delta)}{(1-\beta)}.$$

Note that without learning on the job, the reservation wage component would just be equal to the unemployment benefit component b_0. In the presence of learning ($\mu > 0$), the reservation wage component is lower in high learning environments. In other words, an economy with higher on-the-job human capital growth, μ, has a longer left tail of the firm type distribution and hence a longer part of the job ladder with low quality jobs. This mechanism was first emphasized by Rosen (1972) and is present in our framework as well.

4.3 Firm problem

Firms offer wage contracts $w_0(p)$ that maximise total firm profits. Total profits of the firm are simply obtained as the product of per-worker profits $y - w_0(p)$, which are constant within the firm due to the piece-rate policy, and firm-size $l(w_0)$. Formally, the firms' profits are hence expressed as:

$$\pi(p) = \max_{w_0} \left(y - w_0(p)\right) l(W).$$

Firm size is a function of the job finding rate, λ, and the separation rate, δ, given some constant A such that:

$$l(W) = \frac{A}{\left[\delta + \lambda \overline{F}\left(w_0\right)\right]^2}.$$

The contract distribution must in turn satisfy the equilibrium condition $\Gamma\left[p\right] = F(w_0)$. As shown in appendix section B.2, this implies the equilibrium condition

$$\frac{p - w_0(p)}{\left[\delta + \lambda \overline{\Gamma}\left(w_0(p)\right)\right]^2} - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda \overline{\Gamma}\left(x\right)\right]^2} dx. \tag{3}$$

Note that no firm will offer a wage below the reservation wage component, such that profits at the lower productivity bound are equal to zero. This pins down the effective lower level of productive firms in the economy $\underline{p} = \theta^R = b_0 + \mu\frac{\beta(1-\delta)}{1-\beta}$ We can rewrite equation B.5 to obtain the wage equation as:

$$w(p,h) = h + p - \left(1 + k(1-\Gamma(p))\right)^2 \left(\int_{\underline{p}}^{p} \frac{1}{\left(1 + k(1-\Gamma(x))\right)^2} dx + \pi(\underline{p})\right)$$

using $k = \lambda/\delta$. The wage equation is composed of a worker specific component, h, and a firm specific component, $p - \left(1 + k(1-\Gamma(p))\right)^2 \left(\int_{\underline{p}}^{p} \frac{1}{(1+k(1-\Gamma(x)))^2} dx + \pi(\underline{p})\right)$. The latter is composed of two terms: the first part is increasing in firm productivity, and represents the direct contribution of productivity to wages; the second part is decreasing in firm productivity and represents the rent share component. The literature has discussed that more productive firms have larger monopsony power by facing fewer competition from other firms (cf. Bontemps et al. (2000)). The extend of this monopsony power depends on labor market opportunities, as given by the distribution function $\Gamma(p)$, and labor market frictions, represented by the parameter k.

4.4 Comparative Statics

The model allows us to understand the intricate interrelation between learning rates, labor market opportunities and labor market frictions on the one hand and the age-earnings profile of workers on the other hand. We can distinguish a direct and an indirect effect of these factors on the age-earnings profile. Directly, better access to opportunities such as through a higher job finding rate λ, lower exogenous separation rate δ or a higher share of high productivity firms (long tail of $\Gamma(p)$) increases the average quality of a worker's firm at a given age. Higher learning rates on the job also directly increase the steepness of the age-earnings profile. All else equal, these factors should improve the age-earnings profile of workers. Indirectly, the wage setting protocol, and in particular the relationship between wages and firm productivity, varies with these opportunities as firms adapt their wage offers to their surroundings.

We study this in detail in section B.3 in the appendix, using a functional form assumption for $\Gamma(p)$. In particular, we analyze the change of the steepness of the wage-productivity schedule, $\partial w_0/\partial p$, with key parameters of the model. The steepness increases the wage gains that workers can expect from climbing the job ladder. We show that the wage-productivity schedule, $\partial w_0/\partial p$, is steeper at higher job finding rates λ ($\frac{\partial^2 w}{\partial p \partial \lambda} > 0$) and lower exogenous separation rate δ ($\frac{\partial^2 w}{\partial p \partial \delta} < 0$). We also show that the wage-productivity schedule is steeper the higher the lowest productivity level θ^R ($\frac{\partial^2 w}{\partial p \partial \theta^R} > 0$). As we have seen above, higher growth rates on the job μ depress this lowest productivity boundary, such that there is a negative relationship between learning on the job and the steepness of the wage-productivity schedule in this model. In other words, while higher rates of on-the-job learning increase the steepness of the age-earnings profile directly, they indirectly lower the effect of job market opportunities on the age-earnings profile.

These insights strengthen the motivation to quantify the relative importance of each of these channels for the steepness of the age-earnings profile. We will turn to this in the next section.

5 Empirical Implementation

We are interested in analyzing the age-earnings trajectory for up to 15 years after labor market entrance, which coincides with the period of steepest earnings growth in all studied economies[17]. In the following, we show how we bring the theoretical model to the data. First, we describe parametrizations and define the calibration procedure in section 5.1. We then present estimation results of the model in section 5.2. Using these estimates, we show counterfactual exercises in section 5.3.

5.1 Identification Procedure

Calibration The parameters of the model are composed of three groups: i) the labor market parameters λ, δ; ii) the learning-on-the-job parameter μ; and iii) the firm productivity distribution parameters describing $\Gamma(p)$. For the productivity distribution, we will assume a Pareto distribution with shape parameter α and scale parameter p_0 such that $\Gamma(p) = 1 - \left(\frac{p_0}{p}\right)^{\alpha}$, following Ozkan et al. (2023) and Hubmer (2018). While the scale parameter shifts the lower end of the distribution, the shape parameter governs the right tail of the distribution, with high values implying a lower weight at the right tail of the distribution. Intuitively speaking, extraordinary firm productivities at a high frequency require a low shape parameter α[18]. To set p_0, we are guided by the model. We assume that the lowest productivity firm makes zero profits, such that $\underline{p} = b_0 + \mu\frac{\beta(1-\delta)}{1-\beta}$. We set β consistent with an annual capital return of 10%. To pin down the firm distribution, we are hence identifying the parameter b_0 and α. For estimation of the labor market parameters, we leverage the following two equations. First, we leverage the flow equation for unemployment. In this model, the inflow of employed workers into unemployment through displacement at rate $(1-u)\delta$ and the outflow from unemployment through job finding at rate λu holds a balance such that:

$$\frac{\lambda}{\delta} = \frac{1-u}{u} = k.$$

Second, we use the expected value of the separation rate, denoted $E[qr(w)]$, which can be recovered in closed form due to the functional form assumption for $\Gamma(p)$. Denote with $g(w)$ the distribution of wage components across workers. In Appendix section C, we show that we can obtain the average separation rate as follows:

$$E[qr(w)] = \delta + \lambda \int (1 - F(w))g(w)dw = \delta(1 - k + (1 + k)\log(k + 1)).$$

This allows us to deduce both labor market parameters from the data. For estimation of the learning parameter μ, we leverage the fact that stayer wage growth is driven solely by

[17]This focus also allows us to be consistent with the age coverage in the NLSY.

[18]Similarly, for $\alpha > 0$, the variance and mean of firm productivity decreases in α.

learning on the job,

$$\Delta w = \mu,$$

such that we can pin down the human capital accumulation parameter. While the labor market parameters and the human capital accumulation parameter can hence be recovered from the data directly, we use the simulated method of moments to pin down the remaining parameters α and b_0. To do so, we target the average wage changes at experience level 5-10 and 10-15 compared to age group 0-5 and minimize the squared percentage distance between these moments and the simulated data. In summary, we use the unemployment rate, the exit rate, the stayer wage growth rate and the average wage growth between age groups 0-5, 5-10 and 10-15 for estimation. Table 2 shows these estimation targets.

Table 2: Estimation Targets

	u	$1-qr$	$E[w_{5/10}] - E[w_{0/5}]$	$E[w_{10/15}] - E[w_{0/5}]$
United States	0.117	0.084	48.26	90.26
Brazil	0.253	0.035	33.79	62.25
Colombia	0.162	0.127	16.24	39.33

Notes: This table presents the estimation targets used for estimation. u denotes the unemployment rate, $1-qr$ denotes the worker separation rate and $E[w_{10/15}] - E[w_{0/5}]$ and $E[w_{5/10}] - E[w_{0/5}]$ denote the change in wages in age group 30/35 and 25 to 30 as compared to the age group 20-25.

Note that we do not need to pin down the initial worker skill distribution as these do not affect the changes in average wages between years in our model. Our model hence relies on the estimation of relative moments alone.[19]

Implementation Details To ensure data consistency, and given the narrower age range available in the NLSY dataset, we estimate the model based on the first three age categories: 0-5; 5-10; and 10-15. These categories correspond to the steepest part of the age-earning profile in all three data sets and hence have the highest potential impact on workers' earnings over a lifetime.

[19]Our estimation aims at contrasting the systematic change in average wages with age and the contributions of such systematic changes due to human capital evolution versus changes due to the job ladder. This is fundamentally different from estimating the variance contributions of worker and firm components in residualized wages, where systematic changes due to age are typically removed. An analysis of the variance contributions (as in Ozkan et al. (2023) or Guvenen et al. (2021) for the US) is beyond the scope of this paper and would require data moments that cannot be reliably obtained within any of our labor force surveys or in reliable quality across our sample data sets.

We compute exit rates for all three countries identifying workers who stayed and switched jobs. In the case of the United States, we use the job identifiers of the NLSY to categorize workers as either job stayers or movers. In the Brazilian data, we make use of the panel dimension to determine if a worker remains with the same employer for two consecutive periods. In the Colombian dataset, we leverage firm identifiers to establish the stayer status of workers.

5.2 Parameter estimates and model fit

Table 3 shows the estimation results. We present estimates of the two labor market parameters ,λ, δ, the parameters determining the Pareto distribution of firm productivity, α, b_0, and the human capital growth rate μ.

Table 3: Estimation Results

	Productivity Distribution		Labor Market Parameters		Human Capital
	Shape (α)	Scale (b_0)	Job-Exit (δ)	Job-finding (λ)	Growth (μ)
United States	1.08	2.39	0.007	0.053	0.051
Brazil	1.10	3.83	0.010	0.030	0.039
Colombia	2.14	1.57	0.018	0.093	0.021

Notes: This table presents the estimation results. α denotes the shape parameter of the Pareto distribution $\Gamma(p)$, b_0 determines the scale parameter of the Pareto distribution $\Gamma(p)$, δ denotes the exogenous separation rate, λ denotes the job finding rate and μ denotes the rate of learning-on-the-job.

The estimated human capital growth rate, μ, is higher in the United States than in the other two samples, consistent with the hypothesis that on-the-job training is the main driver of differences in income profiles across countries. Exit rates, on the contrary, are higher in Colombia and Brazil. We observe also higher unemployment rates in these two countries which is consistent with higher job-exit rates. Lastly, the parameter estimates indicate that job-finding rates are higher in Colombia compared to the United States. These two observations align with the findings of Donovan et al. (2023), who established a negative correlation between job-finding and job-exit rates with economic development.

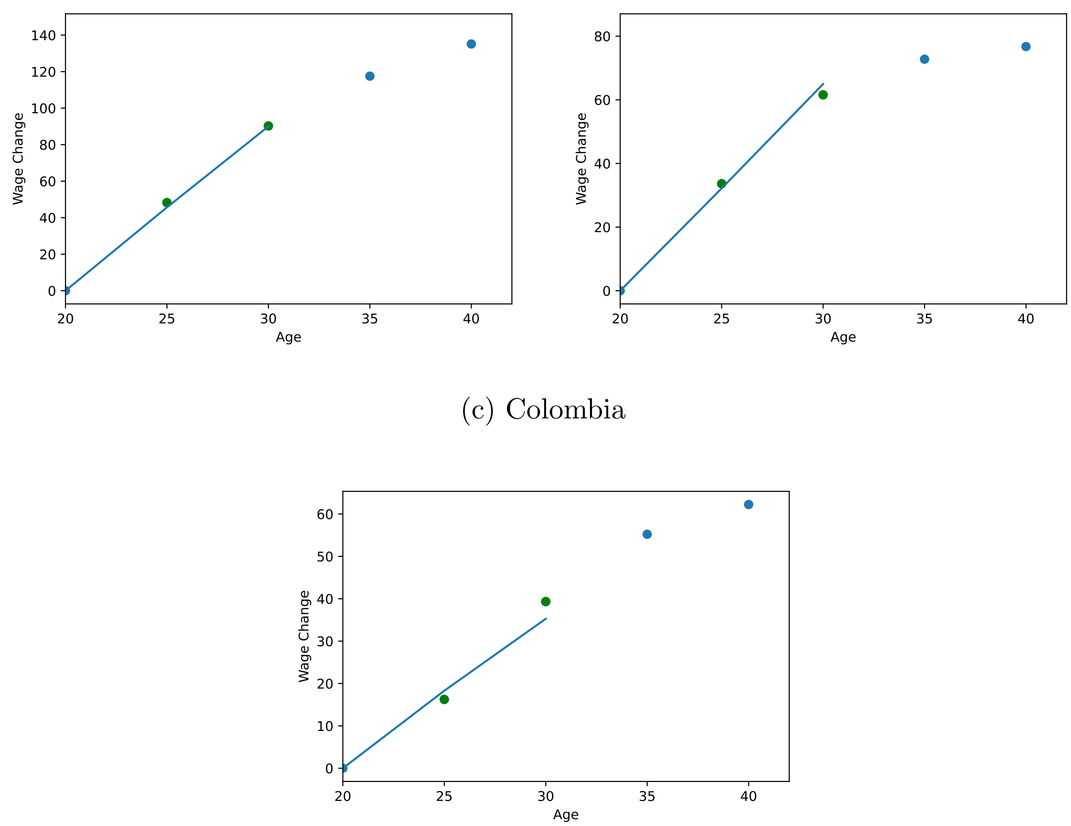
Using the life-cycle wage profile, we pin down the firm distribution shape parameter α and the scale parameter b_0, which pin down the firm distribution scale parameter p_0. We observe striking differences across countries between these parameters. Brazil and Colombia both feature higher shape parameter α than the US estimates, which is indicative of a smaller tail of the Pareto distribution. Figure 4 shows the data together with the estimation results. The model achieves a good fit for the targeted moments of the life-cycle profile.

Figure 4: Moments and Estimation in Comparison

(a) United States

(b) Brazil

(c) Colombia



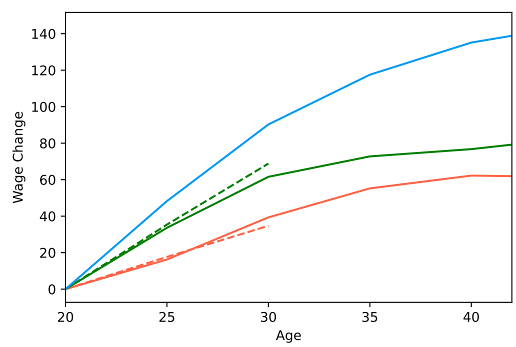
Notes: These figures show the average wage relative to the wage at age group 20-25 for empirical (dots) and simulated data (line). Data for the USA comes from PSID, data for Brazil from PNADC, and data for Colombia from the social security records.

5.3 Counterfactual Analysis

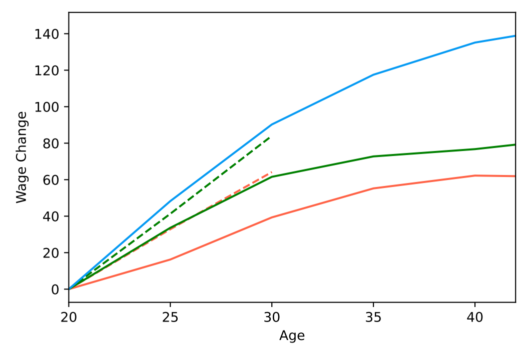
We use the parameter estimates to consider counterfactual scenarios for the life-cycle wage profile across countries. Specifically, we study separately the impact of: i) labor market opportunities; ii) learning on the job; and iii) labor market frictions. We vary one set of parameters at a time, while holding the remainder constant. Results are shown in Figure 5. Each plot varies one aspect of the parameter space for Brazil (in green) and Colombia (in orange). Counterfactual estimates are then shown as dashed lines in the respective color, green for Brazil and orange for Colombia. The U.S. baseline estimate is shown as solid blue line.

Figure 5: Counterfactual Exercise

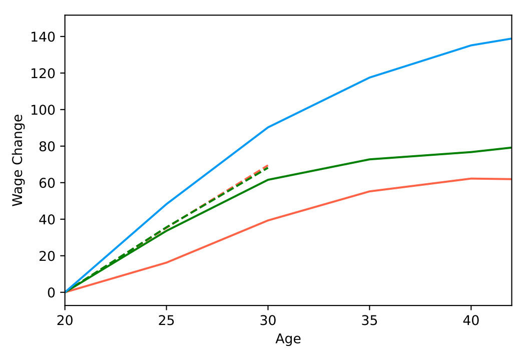
(a) Labor market parameters



(b) On-the-Job Growth



(c) Firm Distribution



Notes: These figures show the average wage relative to the wage at age group 20-25 for both empirical and counterfactually simulated data. Data for the USA comes from PSID, data for Brazil from PNADC, and data for Colombia from the social security records. Each plot varies one aspect of the parameter space for Brazil (green) and Colombia (orange) compared to US estimates (blue). Counterfactual estimates are shown as dashed lines.

Figure 5a shows the life-cycle profile when adjusting the labor market parameters λ, δ to the US level. In both cases, the wage profile is almost unchanged. Figure 5b considers the role of human capital accumulation, whereby we assume the US human capital growth rate for both Brazil and Colombia. For Colombia, increasing the human capital growth rate to the US level would increase life-cycle wage growth to about the factual Brazilian level, while for Brazil life-cycle wage growth would approach the US level. Finally, figure 5c considers an adjustment of the firm type distribution. Specifically, we set the shape parameter α and the parameter b_0 to the US level. These changes have an impact comparable in size to an increase in the human capital growth rate for Colombia, while leaving the Brazilian profile almost unchanged.

These results are summarized in table 4, where we contrast the counterfactual evolution of average wages in Colombia and Brazil with that of the United States. Wage growth in both countries is expressed as the share of wage growth in the United States at age 30-35. We include benchmark measures in the first row of the table indicating that wage growth in Colombia at age 30-35 is equivalent to 43 and 68 percent of the wage growth shown in the United States.

Table 4: Counterfactual Simulation

	Parameters changed to U.S. Level	Relative Wage Growth Colombia	Relative Wage Growth Brazil
Empirical Share		0.43	0.68
Counterfactual :			
Firm Distribution	b_0, α	0.76	0.75
On-the-job Growth Rate	μ	0.71	0.93
Labor Market Parameters	λ, δ	0.38	0.76

Notes: This table presents the empirical value (first row) and three counterfactual estimation results (second to last row), expressed as the share of the wage growth in age bin 30-35 as compared to the value for the US economy.

Better labor market opportunities and on-the-job learning seem to have sizable effects for the Brazilian and Colombian wage profiles. Our results suggest that switching the parameters to match the firm distribution of the United States will increase wage growth from 43 to 76 percent in Colombia and from 68 to 75 percent in Brazil. Moreover, matching the level of on-the-job growth rate will also have sizable effects for both economies, lifting wage growth to 71 percent for Colombia and 93 percent for Brazil. The labor market parameters seem to also have sizable effects for the Brazilian economy, increasing wage growth to 76% of the level of the United States. Altogether, this implies that all three channels are relevant when analyzing wage progression, especially among developing economies.

6 Conclusion

In this paper, we disentangle the impact of labor market opportunities, learning on the job and labor market frictions on differences in age-earnings profiles across the three countries USA, Brazil and Colombia. Empirically, we provide suggestive evidence that human capital growth rates on the job are not the unique and potentially not the most significant driver of differences in age-earnings profiles across these countries. We estimate a random search model and find that differences in labor market opportunities have the largest impact on differences in age-earnings profiles for one of our study countries, followed by differences in learning-on-the job for our second study country.

Our results should be interpreted as lower bounds for the importance of labor market opportunities in shaping age-earnings profiles. Our model interprets wage growth rates of stayers as indicative of on-the-job learning - however, such wage growth could also reflect wage bargaining through outside offers by other firms (Postel-Vinay and Robin (2002)). In such a scenario, on-the-job learning would account for a smaller share of life-cycle wage growth. Moreover, while our model assumes a constant rate of learning on-the-job, it is conceivable that more productive firms offer higher learning rates (Gregory (2020)), which would increase the importance of a long right tail of the firm type distribution.

Our results show that economic development might demand for more variance in firm productivity rather than in less, as might be suggested by a literature that interprets dispersion in productivity as a sign of market frictions (as for instance in Hsieh and Klenow (2014)).

Our results also highlight the crucial role of the job ladder in improving labor market outcomes for workers across countries. Trade policies that allow foreign firms to enter local labor markets might expand the right tail of the job ladder and hence provide attractive labor market opportunities for workers. Trade openness can therefore contribute to steepening the age-earnings profile of workers.

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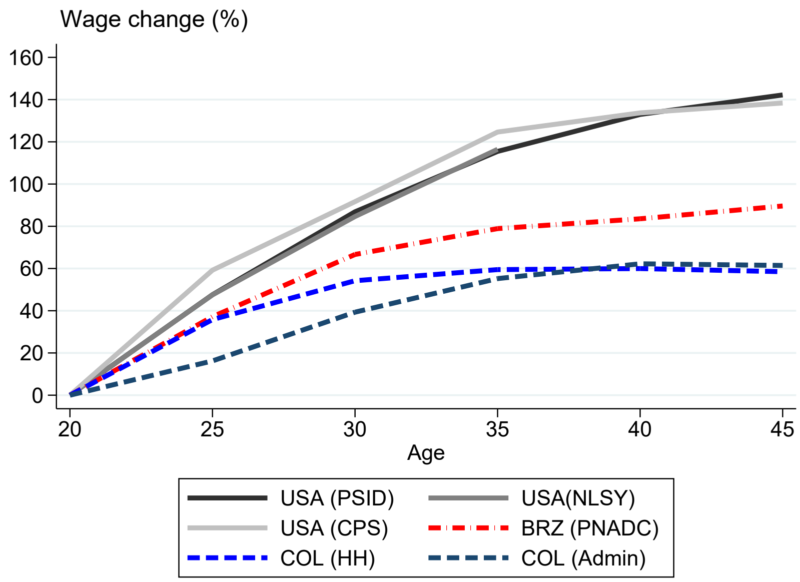
Appendix

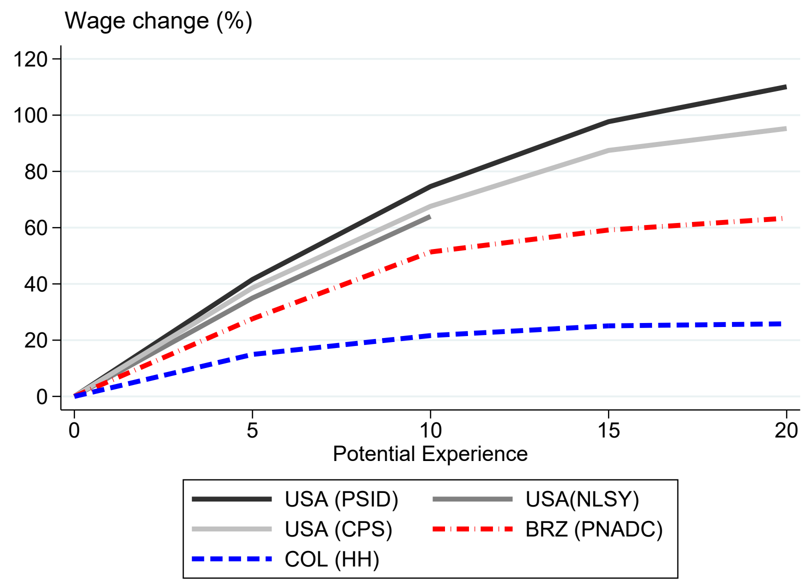
A Appendix Figures and Tables

Appendix Figure A.1: Age Profile of Wages Across Multiple Data sets



Notes: This figure presents growth rates of wages among workers with different age across data sets.

Appendix Figure A.2: Experience Profile of Wages Across Multiple Data Sets



Notes: This graph presents growth rates of wages by level of potential experience. We define potential experience as the lesser of two measurements of duration: the time since reaching the age of 18 or since graduating from the highest level of education.

Appendix Figure A.3: Cross-Regional Suggestive Evidence

(a) Wage Growth of Job Stayers

(b) Share of Job Switchers

(c) Wage Growth of Job Switchers

(d) Share of Employment in Big Firms

Notes: The figures show the cross-regional correlation between relative wage of workers between the ages of 40-44, compared to those between the ages of 20-24, and: a) the average wage growth for stayers; b) the share of job switchers; c) the average wage growth rate for switchers, and d) the share of employees in firms with more than 10 employees. The figures are computed using data from Brazil (PNADC) and the United States (CPS), which are collapsed at the state-country level. Data are weighted by the sum of individual survey weights re-scaled to give equal weight to each country used. All variables are re-scaled between zero and one to enhance comparability across figures.

Appendix Figure A.4: Cross-Country Correlation for European Countries

(a) Wage Growth of Job Stayers

(b) Share of Job Switchers

(c) Wage Growth of Job Switchers

Notes: The figures show the cross-country correlation between relative wage in experience group 15-19, compared to experience group 0-4 years, and: a) the average wage growth for stayers; b) the share of job switchers; and c) the average wage growth rate for switchers. The figures are computed using data from 31 countries in the EU-SILC data and estimated using the model in Equation 1 interacting by measures at the country rather than the region level. Portugal was excluded from the estimation because of inconsistencies in the gross income information, Germany because of small sample size, and Norway because of disparities in the job switching definition. Country-year-Age group cells with less than 50 observations were dropped for the computation of the aggregated country measures. Data are weighted by the sum of individual survey weights re-scaled to give equal weight to each country used.

Appendix Table A.1: Summary Statistics

	Obs. (1)	Mean (2)	SD. (3)	Median (4)	Min. (5)	Max. (6)
A) USA (PSID)						
Age	113,344	38.82	10.91	38.00	20.00	60.00
Female	113,344	0.25	0.44	0.00	0.00	1.00
Log(Monthly Wages)	113,344	7.97	0.94	8.12	4.12	9.68
1(Stayer)	14,175	0.40	0.49	0.00	0.00	1.00
Wage Growth for Stayers	5,229	0.03	0.28	0.01	-2.14	2.22
Wage Growth for Mover	8,178	0.03	0.35	0.02	-2.61	2.30
B) USA (NLSY)						
Age	62,426	27.20	5.14	26.00	20.00	39.00
Female	62,426	0.48	0.50	0.00	0.00	1.00
Log(Monthly Wages)	62,426	7.53	0.84	7.62	4.77	9.23
1(Stayer)	21,397	0.87	0.33	1.00	0.00	1.00
Wage Growth for Stayers	18,406	0.05	0.39	0.03	-3.35	4.46
Wage Growth for Mover	2,991	0.07	0.90	0.03	-3.58	4.46
C) USA (CPS)						
Age	1,578,691	39.41	11.29	40.00	20.00	60.00
Female	1,578,691	0.48	0.50	0.00	0.00	1.00
Log(Monthly Wages)	1,578,691	7.86	0.74	7.89	5.58	9.35
1(Stayer)	357,815	0.96	0.21	1.00	0.00	1.00
Wage Growth for Stayers	342,060	0.02	0.53	0.00	-3.77	3.77
Wage Growth for Mover	15,755	0.03	0.58	0.02	-3.77	3.60
F) Brazil (PNADC)						
Age	6,207,090	37.59	10.78	37.00	20.00	60.00
Female	6,207,090	0.42	0.49	0.00	0.00	1.00
Log(Monthly Wages)	6,207,090	6.23	0.85	6.20	3.53	8.43
1(Stayer)	429,501	0.97	0.16	1.00	0.00	1.00
Wage Growth for Stayers	417,808	0.01	0.43	-0.01	-4.88	4.58
Wage Growth for Mover	11,693	0.01	0.55	0.01	-3.47	3.14
E) Colombia (Social Security)						
Age	69,132,472	38.61	10.05	37.00	20.00	60.00
Female	69,132,472	0.41	0.49	0.00	0.00	1.00
log(Monthly Wages)	69,132,472	5.96	0.63	5.67	-10.27	12.63
1(Stayer)	54,125,512	0.61	0.49	1.00	0.00	1.00
Wage Growth for Stayers	31,999,316	0.02	0.36	0.01	-17.13	16.89
Wage Growth for Mover	17,306,998	0.04	0.54	0.02	-18.76	17.06

Notes: All wages are expressed in 2010 USD, and winsorized in the 1st and 99th percentile. Colombian data have a yearly frequency. Brazilian data have quarterly frequency. The U.S. PSID data set is collected every two years, whereas the U.S. CPS is collected yearly. All samples are restricted to workers between the ages of 20 to 60.

Appendix Table A.2: Determinants of Life-Cycle Wage Growth

	(1)	(2)	(3)	(4)
Share Movers × Age	0.114***			
	(0.021)			
Change in Log Wages of Stayers× Age		0.002***		
		(0.000)		
Change in Log Wages of Switchers× Age			0.001***	
			(0.000)	
Share of Workers in Firms with > 10 Emp.× Age				0.008***
				(0.002)
Observations	2,877,770	2,877,770	2,877,770	2,877,770
Individual FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Age Group FE	Yes	Yes	Yes	Yes

Note: This table presents the results the estimation of equation 1. Survey weights are rescaled to give equal weight to each country. Standard errors clustered at the state level. *** p<0.01, ** p<0.05, * p<0.1

B Mathematical Details

B.1 Derivation Reservation Wage Component

Using integration by parts and incorporating the reservation strategy, we can write the two value functions as:

$$U(h) = b(h) + \beta U(h) + \beta\lambda \int_{\theta^R}^{\overline{w}} W_x \overline{F}(x) dx \tag{B.1}$$

$$W(w_0, h) = w + \beta\lambda \int_{w_0}^{\overline{w}} W_x \overline{F}(x) dx + \beta\delta U(h) + \beta(1-\delta) W(w_0, h') \tag{B.2}$$

Differentiating the worker value function with respect to w_0 yields:

$$W_w(w_0, h) \;\; = \;\; 1 + \beta(1 - \delta - \lambda\overline{F}(w)) W_w(w_0, h').$$

Solving for $W_w(w_0, h)$ we find that[20]:

$$W_w(w_0, h) \;\; = \;\; \frac{1}{1 - \beta(1 - \delta - \lambda\overline{F}(w))}. \tag{B.3}$$

This implies that the option value of search does not depend on a worker's human capital. We pin down the reservation wage component, θ^R, by combining the wage equation in (2) with the value functions in (B.1) and (B.2) and the equation for unemployment benefits. This combination implies that:

$$b_0 - \theta^R \;\; = \;\; \beta(1-\delta)\left(W(\theta^R, h') - U(h')\right) + \beta(1-\delta)\left(U(h') - U(h)\right) \tag{B.4}$$

We proceed to guess and verify an equilibrium with equal reservation wage component for all workers, such that $\partial\theta^R/\partial h = 0$. In this case, using the equation for $U(h)$ together with the result in equation B.3 that $\frac{\partial W_{w_0}}{\partial h} = 0$, we find that

$$(U(h') - U(h)) = \frac{h' - h}{1 - \beta} = \frac{\mu}{1 - \beta}.$$

[20]Note that this result is identical as in Bagger et al. (2014).

Together with the equation for $W(\theta^R, h) - U(h)$, this implies that the reservation wage component is[21]:

$$\theta^R \;=\; b_0 - \mu\frac{\beta(1-\delta)}{1-\beta}$$

B.2 Wage equation

The envelope theorem implies the equilibrium condition

$$\pi'(p) = \frac{1}{\left[\delta + \lambda\overline{F}\left(w_0(p)\right)\right]^2}$$

or

$$\pi(p) - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda\overline{\Gamma}\left(w_0^{-1}\right)\right]^2} dx$$

where $w_0^{-1} = q$ denotes the inverse function to $w_0(p)$. We also denote the first-order condition

$$2(p - w_0(p))\frac{\lambda f(w_0)}{\delta + \lambda\overline{F}\left(w_0\right)} = 1.$$

It follows from the equilibrium condition that

$$\frac{p - w_0(p)}{\left[\delta + \lambda\overline{\Gamma}\left(w_0(p)\right)\right]^2} - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda\overline{\Gamma}\left(x\right)\right]^2} dx. \tag{B.5}$$

B.3 Comparative Statics

We study the change in the slope of the wage-productivity schedule with respect to some parameter ρ, that is $\frac{\partial^2 w}{\partial p \partial \rho}$. To do this, we leverage the first order condition with equilibrium condition $F(w_0(p)) = \Gamma(p)$

$$2(p - w_0(p))\frac{\lambda\gamma(p)}{\delta + \lambda\overline{\Gamma}\left(p\right)} = 1$$

[21] To see that this is indeed an equilibrium, differentiate equation B.4 with respect to h to obtain

$$\frac{\partial\theta^R}{\partial h}\left(-1 - \beta(1-\delta)W_{\theta^R}\right) = \beta(1-\delta)\left(W_h - U_h\right)$$

For $\frac{\partial\theta^R}{\partial h} = 0$ we require that $W_h = U_h$. In this case, $U_h = \frac{1}{1-\beta}$ and $W_h = 1 + \beta\delta U_h + \beta(1-\delta)W_h = \frac{1}{1-\beta}$

using the parametric assumption $\Gamma(p) = 1 - \left(\frac{p_0}{p}\right)^{\alpha}$. By the implicit function theorem,

$$\left(1 - \frac{\partial w}{\partial p}\right) - (\alpha+1)\frac{\delta + \lambda\overline{\Gamma}(p)}{\lambda\gamma(p)} - 1/2 = 0$$

Hence, we know that

$$\begin{aligned}
\frac{\partial^2 w}{\partial p \partial \lambda} &= (\alpha+1)\frac{\delta p^{\alpha+1} p_0^{-\alpha}}{\lambda\alpha} > 0 \\
\frac{\partial^2 w}{\partial p \partial \delta} &= -(\alpha+1)\frac{p^{\alpha+1} p_0^{-\alpha}}{\lambda\alpha} < 0 \\
\frac{\partial^2 w}{\partial p \partial p_0} &= (\alpha+1)\frac{\delta p^{\alpha+1} p_0^{-\alpha}}{\lambda} > 0 \\
\frac{\partial^2 w}{\partial p \partial \alpha} &= (\alpha+1)\left(-\left(\frac{p^{\alpha+1} p_0^{-\alpha}(\delta + \lambda \frac{p_0}{p}^{\alpha})}{\lambda\alpha^2}\right) - \left(\frac{\log(p) p^{\alpha+1} p_0^{-\alpha}(\delta + \lambda \frac{p_0}{p}^{\alpha})}{\lambda\alpha}\right)\right) \\
&- (\alpha+1)\left(\left(\frac{\log(p_0) p^{\alpha+1} p_0^{-\alpha}(\delta + \lambda \frac{p_0}{p}^{\alpha})}{\lambda\alpha}\right) - \left(\frac{p^{\alpha+1} p_0^{-\alpha} \frac{p_0}{p}^{\alpha} \log(p_0/p)}{\alpha}\right)\right) <> 0
\end{aligned}$$

C Exit Rate

A worker's likelihood of exiting a firm with wage component w_0, $qr(w_0)$, is composed of the exogenous likelihood of separating, δ and the likelihood of job mobility. The latter depends on the wage offer w_0 and the wage offer distribution $F(w_0)$, such that any draw from the job offer distribution exceeding w_0 will be accepted (occuring at rate $(1-F(w_0))$). It also depends on the likelihood of job arrivals λ. The compound likelihood of job mobility is hence $\lambda(1 - F(w_0))$. Together, we write the likelihood of exiting as

$$qr(w_0) = (\delta + \lambda(1 - F(w_0)))$$

The expectation of the job exit rate is then

$$\begin{aligned}
E[qr] &= \delta + \lambda \int_{\underline{w}}^{\overline{w}} (1 - F(w)) g(w) dw \\
&= \delta + \lambda \int_{\underline{p}}^{\infty} (1 - \Gamma(p)) g(w(p)) w_p dp \\
&= \delta + \lambda \int_{\underline{p}}^{\infty} \frac{\bar{\Gamma}(p)\gamma(p)(1+k)}{\left(1 + k\bar{\Gamma}(p)\right)^2} \frac{1}{\frac{dw}{dp}} w_p dp
\end{aligned}$$

where we use the fact that the wage offer distribution $F(w)$ is related to the distribution of workers across contracts $G(w)$ (cf. Bontemps et al. (2000)) in

$$1 + kG(w(p)) = \frac{1+k}{1+k\bar{F}(w(p))} = \frac{1+k}{1+k\bar{\Gamma}(p(w))}$$

We deduce[22]

$$\frac{\partial G(w_0(p))}{\partial p} = g(w(p)) = \frac{\gamma(p)(1+k)}{\left(1+k\bar{\Gamma}(p)\right)^2}\frac{1}{\frac{dw}{dp}}$$

Assuming a Pareto distribution with scale parameter $b = \underline{p}$ and shape parameter a with $\bar{\Gamma}(p) = \left(\frac{b}{p}\right)^a$, such that $\gamma(p) = \frac{a}{b}\left(\frac{b}{p}\right)^{a+1}$ we obtain

$$\begin{aligned}
E[qr(w)] &= \delta + \lambda \int_b^\infty \frac{\frac{a}{b}\left(\frac{b}{p}\right)^{2a+1}(1+k)}{\left(1+k\left(\frac{b}{p}\right)^a\right)^2} dp = \delta + \lambda(1+k)ab^{2a}\int_b^\infty \frac{p^{-1}}{(p^a+kb^a)^2}dp \\
&= \delta + \lambda\left((1+k)ab^{2a}\frac{\left(\frac{b^a k}{kb^a+p^a} - \log\left(kb^a+p^a\right) + a\log(p)\right)}{kab^{2a}}\right)\Big|_b^\infty \\
&= \delta + \lambda\left(\frac{(1+k)}{k}\left(\frac{b^a k}{kb^a+p^a} - \log\left(kb^a+p^a\right) + a\log(p)\right)\right)\Big|_b^\infty \\
&= \delta + \lambda\left(\frac{(1+k)}{k}\left(\frac{\frac{b}{p}^a k}{k\frac{b}{p}^a+1} + \log\left(\frac{1}{k\left(\frac{b}{p}\right)^a+1}\right)\right)\right)\Big|_b^\infty \\
&= \delta - \lambda\left(\frac{(1+k)}{k}\left(\frac{k}{k+1} + \log\left(\frac{1}{k+1}\right)\right)\right) = \delta - k\delta\left(\frac{(1+k)}{k}\left(\frac{k}{k+1} + \log\left(\frac{1}{k+1}\right)\right)\right) \\
&= \delta(1 - k + (1+k)\log(k+1))
\end{aligned}$$

Note that this expression is independent of the firm-type distribution.

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[22] From

$$\begin{aligned}
1 + kG(w) &= \frac{1+k}{1+k\bar{F}(w)} \\
\left(1+k\bar{F}(w)\right)kG(w) &= (1+k) - \left(1+k\bar{F}(w)\right) = kF(w)
\end{aligned}$$

Given equilibrium constraint $F(w(p)) = \Gamma(p(w))$

$$G(w(p)) = \frac{\Gamma(p(w))}{1+k\bar{\Gamma}(p(w))}$$

Hence

$$G'(w(p)) = \frac{\left(\gamma(p)(1+k\bar{\Gamma}(p)) + k\gamma(p)\Gamma(p)\right)}{\left(1+k\bar{\Gamma}(p)\right)^2}\frac{1}{\frac{dw}{dp}} = \frac{\gamma(p)(1+k)}{\left(1+k\bar{\Gamma}(p)\right)^2}\frac{1}{\frac{dw}{dp}}$$